# Design of Rigid-Body Motion

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Keyframing
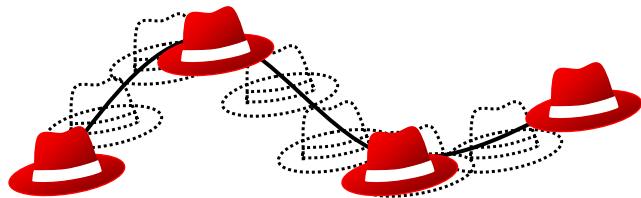
Perfect control

*Tedious*

No realism

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Numerical Simulation

$$F = ma$$
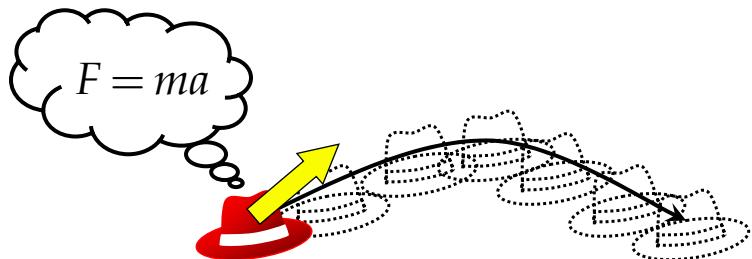
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Numerical Simulation
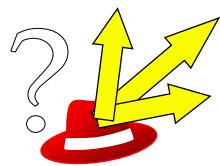
Realistic Motion

Automatic Generation

Difficult to Control

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Rigid-Body Motion
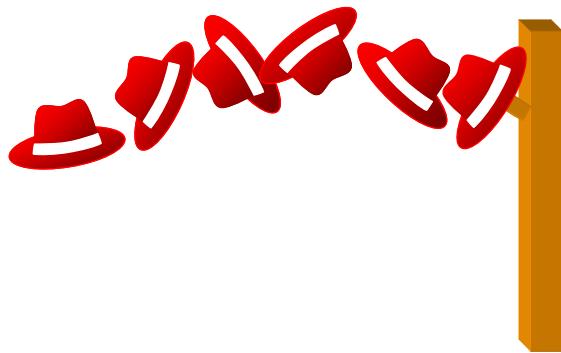
Motion Design

Realistic Motion

Direct Control

Intuitive Design

- Sketch
- Edit
- Act

Outline
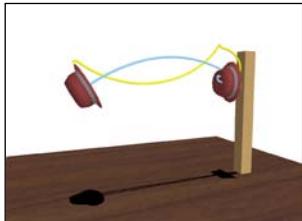
Interactive Manipulation of
Rigid Body Simulations
[J. Popović, Seitz, Erdmann, Z. Popović, Witkin]
SIGGRAPH 2000

Motion Sketching
[J. Popović, Seitz, Erdmann]
Transactions on Graphics 2003

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Related Fields

Boundary Value Problems

   [Ascher, Mattheij, Rusell]

Optimal Control and Estimation

   [Stengel]

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Boundary Value Problem

$$F = ma$$
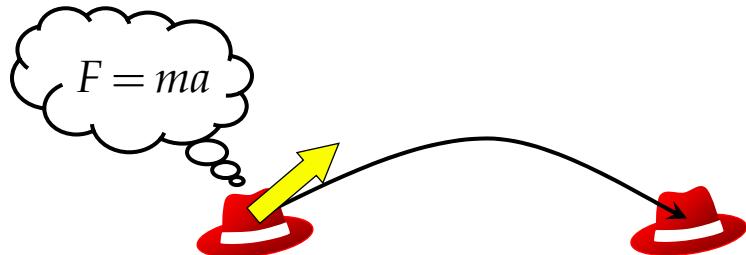
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Related Fields

Boundary Value Problems

[Ascher, Mattheij, Rusell]

Optimal Control and Estimation

[Stengel]

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Related Work

Spacetime Constraints [Witkin, Kass]

Sampling Plausible Solutions to Multi-Body Constraint Problems [Chenney, Forsyth]

Kinodynamic Planning [Canny, Donald, Reif, Xavier]

Randomized Kinodynamic Planning [LaValle, Kuffner]

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Interactive Manipulation
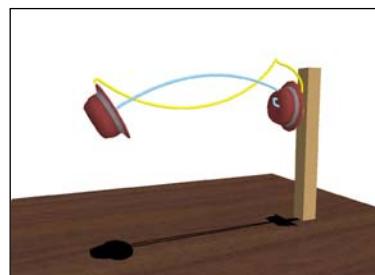
Interactive design
Direct control
Realistic motion

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Simulation Function

initial velocity → simulator → $q(t)$ u

$\underbrace{\qquad}_{u}$ $\underbrace{\qquad}_{\substack{S \\ \text{nonlinear} \\ \text{not continuous}}}$ $\underbrace{\qquad}_{q(t) = S(t, u)}$
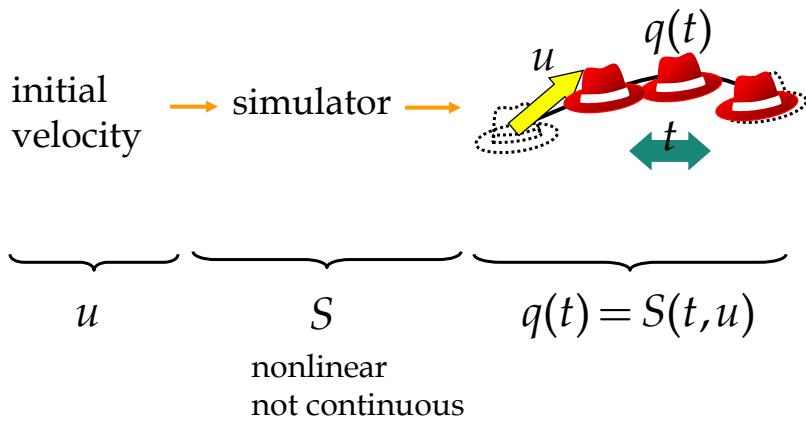
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Simulation Function

Not continuous

$S(t_f, u)$

u

$S(t_f, u_\delta)$
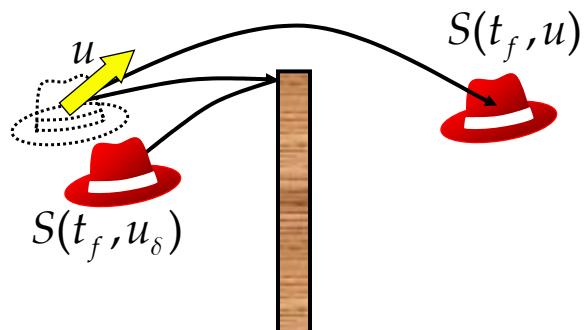
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Simulation Function

Free-flight motion

$$\frac{d}{dt}q(t) = \frac{d}{dt}\begin{pmatrix} x(t) \\ r(t) \\ m\,v(t) \\ I(t)\omega(t) \end{pmatrix} = \begin{pmatrix} v(t) \\ \dfrac{1}{2}\omega(t)r(t) \\ F(t) \\ \tau(t) \end{pmatrix}$$

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Simulation Function

Impacts

$$q^{+} = q^{-} + J(q^{-}, m, I, \varepsilon, n)$$

n
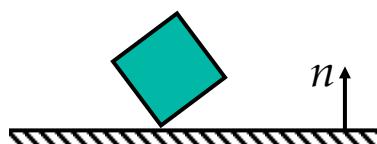
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Differential Control
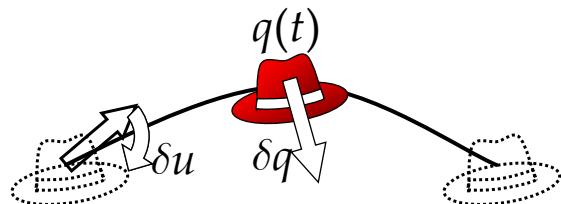
$$q(t) = S(t,u) \quad \Longrightarrow \quad \delta q = \frac{\partial S(t,u)}{\partial u}\, \delta u$$

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Interactive Loop

u | $S(t,u)$
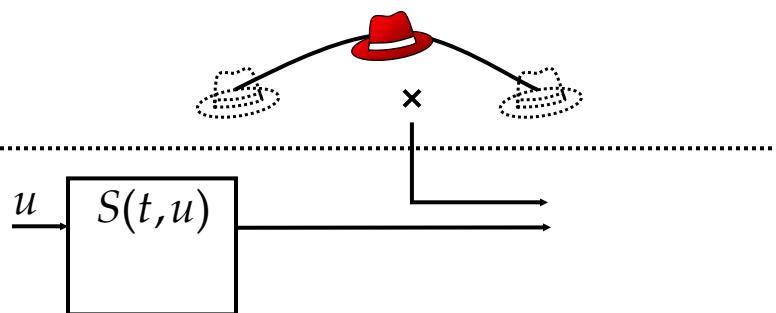
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Interactive Loop

δq

$S(t, u)$ → $-$ → δq

Interactive Loop

δu δq

u | $S(t, u)$
$\partial S / \partial u$ → $-$ → δq

$\dfrac{\partial S}{\partial u}$
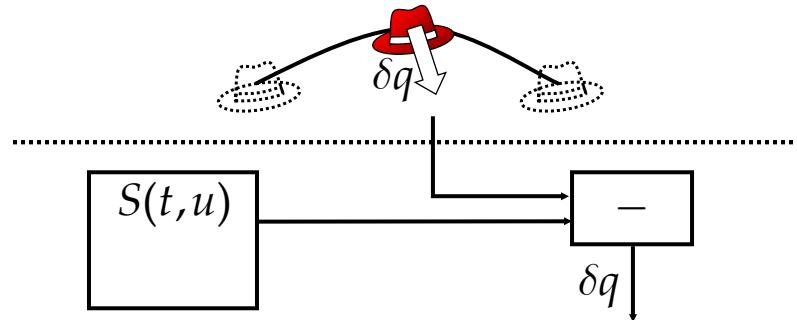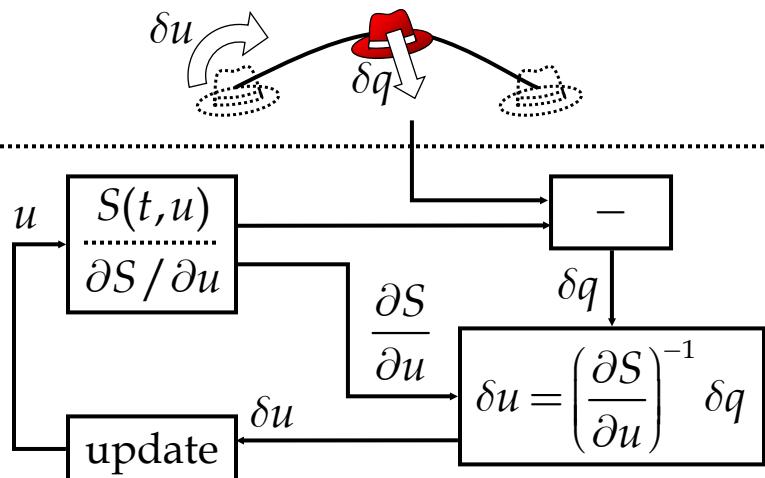
$\delta u = \left(\dfrac{\partial S}{\partial u}\right)^{-1} \delta q$

update ← δu

Derivative Computation

$$\frac{S(t,u)}{\partial S / \partial u}$$
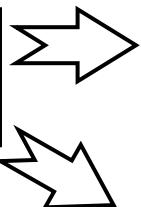
Rigid-body simulator

[Baraff]

Automatic differentiation

- analytic derivatives
- numeric integration

Leibnitz Rule

Differential Update

$$\delta u = \left(\frac{\partial S}{\partial u}\right)^{-1} \delta q$$
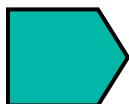
$$\min_{\delta u} \quad \delta u^T M \, \delta u$$

$$\text{s. t.} \quad \delta q = \frac{\partial S}{\partial u} \delta u$$

More Control

velocity
surface
gravity
...

$\longrightarrow$ simulator $\longrightarrow$

$$u_3$$

$$u_0$$

$$q(t)$$

$$u_1$$

$$u = \begin{pmatrix} u_0 \\ \vdots \\ u_3 \end{pmatrix}$$

$$S$$

$$q(t) = S(t,u)$$

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More Constraints

velocity
surface
gravity
...

$\longrightarrow$ simulator $\longrightarrow$

$$\delta q_1$$

$$\delta q_0$$

$$u$$

$$S$$

$$\delta u = \begin{pmatrix} \dfrac{\partial S(t_0,u)}{\partial u} \\ \dfrac{\partial S(t_1,u)}{\partial u} \end{pmatrix}^{-1} \begin{pmatrix} \delta q_0 \\ \delta q_1 \end{pmatrix}$$

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More Bodies

velocity
surface
gravity
…

$\longrightarrow$ simulator $\longrightarrow$ $q_0(t)$ $q_1(t)$

$\underbrace{\qquad}_{u}$ $\underbrace{\qquad}_{S}$ $\underbrace{\qquad}_{\begin{pmatrix} q_0(t) \\ q_1(t) \end{pmatrix} = S(t, u)}$

More Motions

$$\frac{S(t, u)}{\partial S / \partial u}$$

$\Longrightarrow$
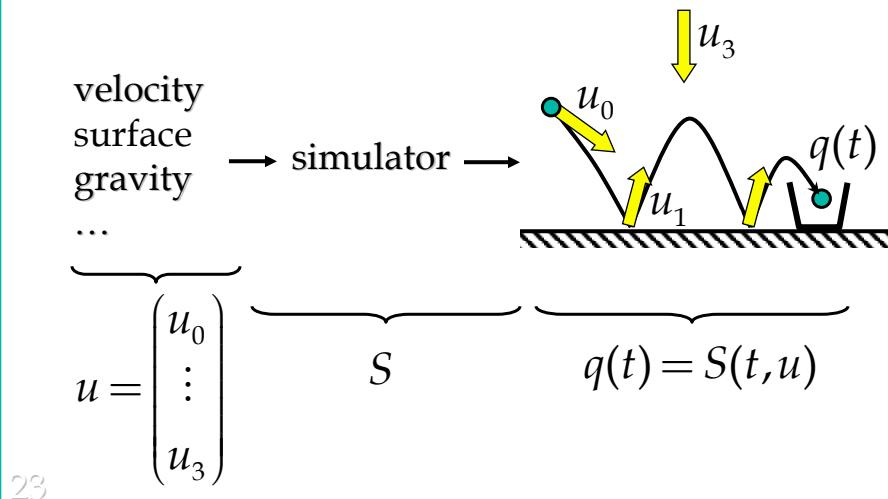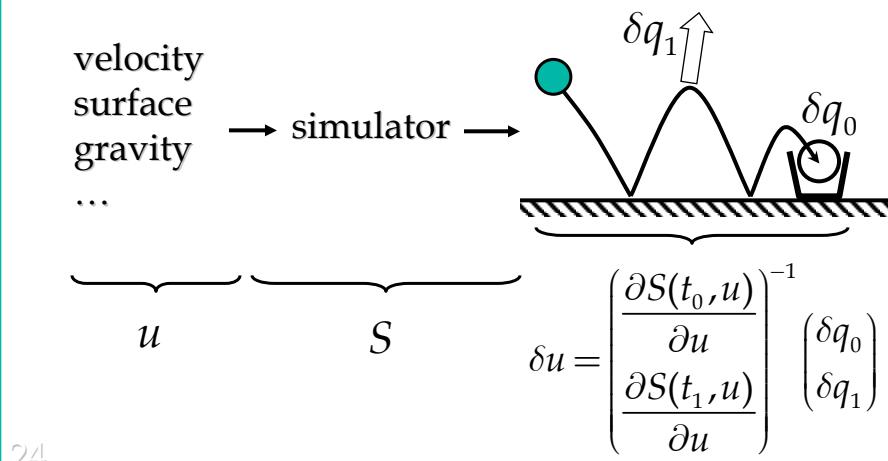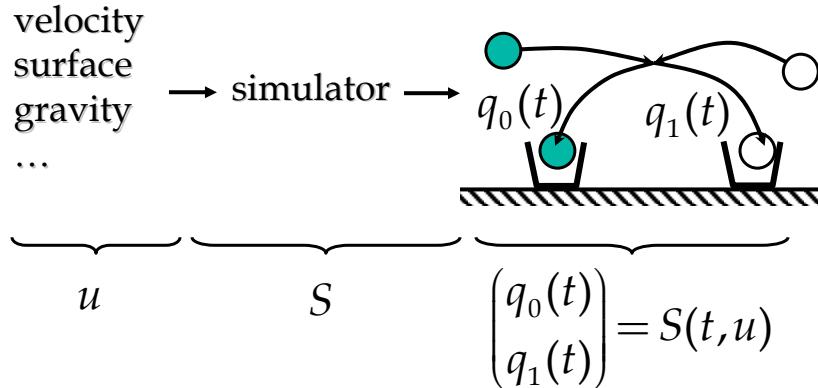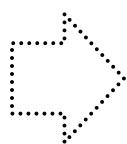
Particles

Linked Rigid Bodies
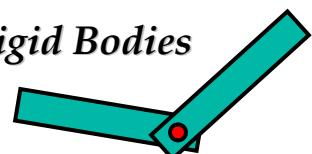
*Flexible Bodies*

Limitations

Improbable motions

- many constraints few degrees of freedom

*Complex physical behaviors*

- time-consuming physical simulations

Sliding and static contacts

- complex linear complementarity formulation

*Discrete/continuous search*

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Motion Sketching

Sketch ⟹ *Realistic Motion*

type of motion

specific goals

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Motion Sketching

$q(t) = S(t, q_0)$

q_0

$q(t)$

$$\min_{q_0} \quad E[q(t), \circ] \qquad \text{type}$$

$$\text{subject to} \quad C_{\otimes}[q(t)] = 0 \quad \text{goals}$$
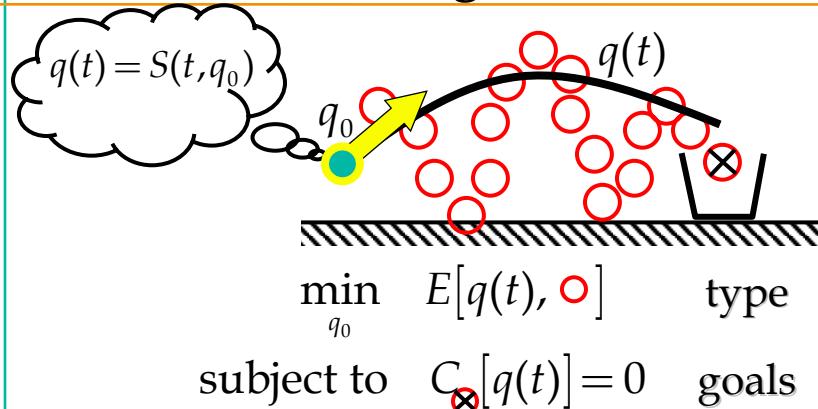
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Motion Sketching

q_0

$$\min_{q_0} \quad E[q(t), \circ] \qquad \text{type}$$

$$\text{subject to} \quad C_{\otimes}[q(t)] = 0 \quad \text{goals}$$
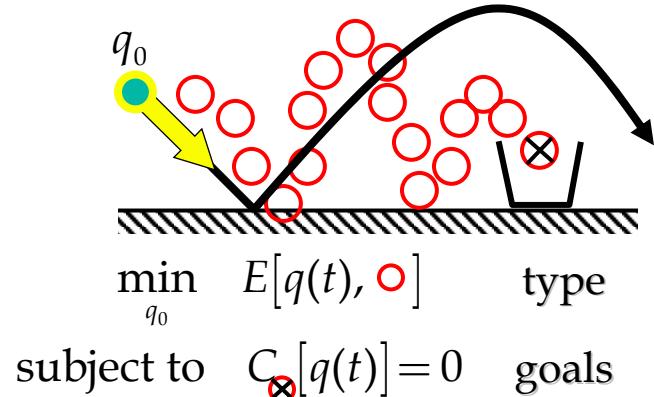
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Motion Sketching

Single Shooting
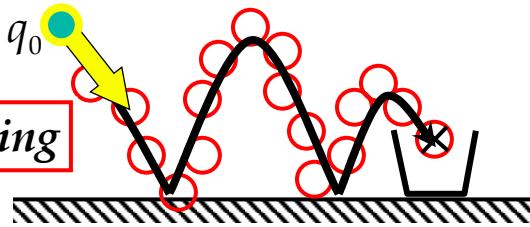
$$\min_{q_0} \quad E\big[q(t), \circ\,\big] \qquad \text{type}$$

$$\text{subject to} \quad C_\otimes\big[q(t)\big] = 0 \quad \text{goals}$$

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Multiple Shooting
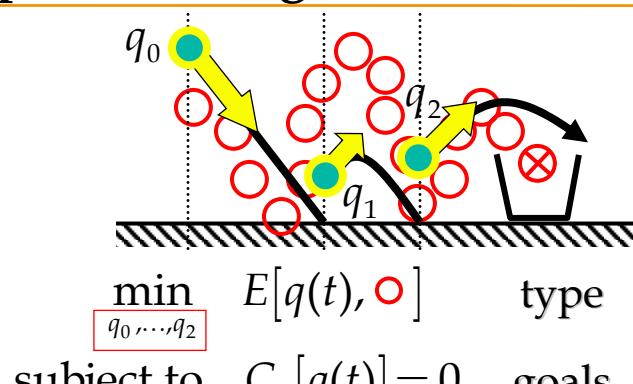
$$\min_{q_0,\dots,q_2} \quad E\big[q(t), \circ\,\big] \qquad \text{type}$$

$$\text{subject to} \quad C_\otimes\big[q(t)\big] = 0 \quad \text{goals}$$

$$C_c(q_0,\dots,q_2) = 0 \quad \text{physics}$$

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Multiple Shooting

$$\min_{q_0,\ldots,q_2} \quad E\big[q(t), \circ\big] \qquad \text{type}$$

$$\text{subject to} \quad C_{\otimes}\big[q(t)\big] = 0 \quad \text{goals}$$

$$C_c(q_0,\ldots,q_2) = 0 \quad \text{physics}$$

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Sliding Time Grid
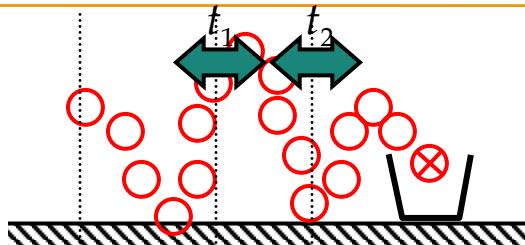
$$\min_{q_0,\ldots,q_2} \quad E\big[q(t), \circ\big] \qquad \text{type}$$

$$\text{subject to} \quad C_{\otimes}\big[q(t)\big] = 0 \quad \text{goals}$$

$$C_c(q_0,\ldots,q_2) = 0 \quad \text{physics}$$

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Multiple Shooting

q_0
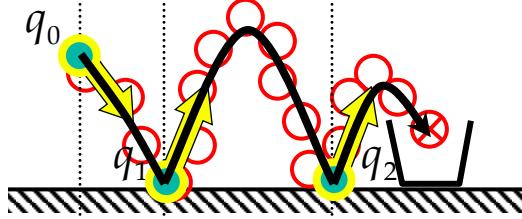
$$\min_{q_0,\ldots,q_2} \quad E\big[q(t), \circ\,\big] \qquad \text{type}$$

$$\text{subject to} \quad C_\otimes\big[q(t)\big] = 0 \quad \text{goals}$$

$$C_c(q_0,\ldots,q_2) = 0 \quad \text{physics}$$

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Summary

Realistic motion

Direct control

Intuitive design

- Edit
- Act

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Directions for Future Work

Sliding and static contact

Physically infeasible sketches

Discrete/continuous search

Sketching Interfaces

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More Design Problems

Human and Animal Locomotion

- Character Animation
- Robotics
- Biomechanics

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More Design Problems

Physical Motion

- Cloth
- Fluids
- Sculpting clay

Non-physical Motion

- Digital clay
- Cartoon
- Claymation

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